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# TOURISM, TRADE, EXTERNALITIES, AND PUBLIC GOODS IN A THREE-SECTOR GROWTH MODEL

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**Abstract**
The purpose of this study is to introduce tourism, externalities, and public goods to a small-open growth with endogenous wealth and public goods supply. We develop the model on the basis of the Solow-Uzawa growth model, the neoclassical neoclassical growth theory with externalities, and ideas from tourism economics. The economy consists of three – service, industrial, and public - sectors. The production side is based on the traditional growth theories, while the household behavior is described by an alternative utility function proposed by Zhang. We introduce endogenous land distribution between housing and supply of services. The industrial and service sectors are perfectly competitive subject to the government's taxation. The public sector is financially supported by the government. We introduce taxes not only on producers, but also on consumers' incomes from wage, land, and interest of wealth, consumption of goods and services, and housing. We simulate the motion of the national economy and show the existence of a unique stable equilibrium. We carry out comparative dynamic analysis with regard to the rate of interest in the global market, the total productivity of the service sector, tax rate on the service sector, tax rate on consumption of services, human capital, the propensity to consume services, and the impact of public services on the productivity of the industrial sector. The comparative dynamic analysis provides some important insights into the complexity of open economies with endogenous wealth, public goods, and externalities.

Keywords: tourism, taxes, public goods, price elasticity of tourism, wealth accumulation.

*Jel Classification:* H23; O41; Q56

## INTRODUCTION

The purpose of this study is to deal with dynamic interactions between economic growth, economic change, tourism, and trade with externalities and endogenous public goods. The model is a synthesis of a few approaches in economic theories. A main concern of this study is how tourism interacts with national economic development and economic

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structural change. Tourism is closely related to domestic economic conditions as well as global economic development. It is obvious that tourism is increasingly becoming important part of national economies. The export income of international tourism is the fourth after fuels, chemicals, and automotive products. As mentioned by Copeland (2012) tourism accounts for 6 per cent of global exports overall and thirty per cent of global exports of services. Tourism needs special attention in studying issues related to international trade because tourism is different from what is called tradable goods in traditional trade theory. Tourism goods such as monuments of national heritage, historical sites, beaches, and hot springs, are not-tradable in the traditional trade theory as one has to travel to the location in order to consume them. Tourism converts non-traded goods into tradable ones. Tourism affects local economies in different ways. Tourism uses national resources such as labor, capital and housing and thus may make these resources less available for other sectors of the economy. Tourism also generates income which may be used to develop other economic activities. Tourism has been analyzed by economists from different aspects within different theoretical frameworks (e.g., Sinclair and Stabler 1997; Luzzi and Flückiger 2003; Dritsakis 2004; Durbarry 2004; Hazari and Sgro 2004; Briedenhann and Wickens 2004; Katircioglu 2009; Hazari and Lin 2011; and Ridderstaat, Croes, and Nijkamp 2014). Chao *et al.* (2009) demonstrates that most of these economic studies of tourism are conducted within static frameworks (see also, Zeng and Zhu 2011; Corden and Neary 1982; Copeland 1991). Dynamic issues related to tourism cannot be properly examined within static frameworks. Dwyer, Forsyth and Spurr (2004) discuss the need for dynamic general equilibrium modelling when studying tourism and its interaction with the rest economy. Blake, Sinclair, and Soria (2006) also address the issue. This study tries to introduce tourism to growth theory with endogenous wealth and public goods.

Dynamic interdependence between economic growth and public investment is a main topic in economic theory (see, for instance Barro 1990, Turnovsky 2000, 2004). Some of these studies have taken account of effects of congestion of public goods with different fiscal policies on economic growth theory (for instance, Glomm and Ravikumar 1997, Gómez 2008). The purpose of this study is to introduce public goods, externalities and congestion into a growth model with tourism. The approach is based on Zhang's growth model with public goods (Zhang 2014). We show that externalities and congestion may have different effects on the growth process. This study analyzes the effects of fiscal policies in an economy with public and private capital. Another unique feature of this study is that we consider different taxes. There are different ways of taxation in different analytical frameworks and the government's income may be spent in different ways (Jha 1998, 2003). This study assumes that the government finances the public sector by imposing taxes on the outputs of the firms, and the household's capital income, labor income, land income, and consumptions. The government's income is spent only on supplying public goods. The public sector gets the government's income and supplies public goods.

After many efforts by economists, it has become clear that it is not easy to formally model economic development and dynamics of tourism with public goods on microeconomic foundation. This study will build an economic growth model with public goods and dynamics of tourism. The economic production and market aspects are based on the neoclassical growth theory. The models in the neoclassical growth theory are extensions and generalizations of the pioneering works of Solow (1956). As far as the economic structure is concerned, this study is influenced by Uzawa (1961), who made an extension of Solow's one-sector economy by a breakdown of the productive system into two sectors using capital and labor (see also, Diamond 1965; Stiglitz 1967; Drugeon and Venditti 2001). In the Uzawa two-sector growth model, one sector produces industrial goods and the other consumption goods. Our growth model with economic structural with public goods and tourism is developed within the Uzawa analytical framework. We introduce public goods and tourism into the Uzawa two-sector model. To properly deal with tourism, we accept an analytical framework for small open economies. In the literature of tourism economics, almost all the models are built within a small open economic framework (e.g., Zeng and Zhu 2011). There is a large number of the literature on economics of open economies (e.g., Obstfeld and Rogoff 1996; Lane 2001; Kollmann 2001, 2002; Benigno and Benigno 2003; Galí and Monacelli 2005; Uy, Yi, and Zhang 2013; and Ilzetzkia, Mendozab, and Véghc 2013). We follow this tradition in dealing with dynamic interdependence between economic structural change, public goods, tourism, and wealth accumulation. Our approach is different from the traditional approaches in the neoclassical growth theory. There are three main frameworks in modeling household behavior in economic growth theory with capital accumulation. The Solow model is the starting point for almost all analyses of economic growth (Solow 1956). The Solow model does not provide a mechanism of endogenous savings. Another important approach is the so-called representative agent growth model based Ramsey's utility function (Ramsey 1928). Cass and Koopmans integrated Ramsey's analysis of consumer optimization and Solow's description of profit-maximizing producers within a compact framework (Cass 1965; Koopmans 1965). One of the problems of this approach is that it makes the analysis intractable even for a simple economic growth problem. Another approach in economic modeling is the so-called OLG approach (Diamond 1965, Samuelson 1959). The approach is a discrete version of the continuous Ramsey approach (Azaridias 1993). This study will model behavior of households with an alternative approach proposed by Zhang in the early 1990s (Zhang 1993). The model in this study is an extension of a growth model with tourism by Zhang (2012). The introduction of tourism to growth theory is influenced by Chao *et al*. (2006). A main different between our approach and the model by Chao *et al.* is that this study is based on an alternative utility function proposed by Zhang (1993). This model is different from Zhang's 2012 model is that this study introduces endogenous public goods and endogenous time distribution basing on Zhang's 2014 model with public goods and endogenous time distribution for a closed national economy (without tourism). Zhang's 2012 growth model with tourism is a special case of the model in this study. We examine the dynamic effects of different government policies. In examining behavior of the model, our attention is focused on the numerical simulations of a calibrated economy. We highlight the dynamic effects of exogenous parameter changes. Section 2 defines the basic model. Section 3 shows how we solve the dynamics and simulates the model. Section 4 examines effects of changes in some parameters on the economic system over time. Section 5 concludes the study. The appendix proves the main results in Section 3.

## 1. THE GROWTH MODEL WITH PUBLIC GOODS AND TOURISM

This section develops a small-open three-sector growth model with endogenous wealth and public goods. We consider that the open economy can import goods and borrow resources from the rest of the world or exports goods and lend resources abroad. Our model is an integration of the basic features of a few well-known models in the literature of economic growth. They include the Solow growth model, the Uzawa two-sector growth model, neoclassical growth models with elastic labor supply, and the growth models with tourism. There is a single good, called industrial good, in the world economy and the price of the industrial good is unity. Like in Chao *et al.* (2009) and Zhang (2012), we consider the economy produces two goods: an internationally traded good (called industrial good) and a non-traded good (called services). It should be noted that in Brock (1988) products of the economic activities of the small-open economy are divided traded and non-traded goods. This study extends the traditional model by taking account of tourism, endogenous time distribution and public goods supply. Domestic households consume both goods. Foreign tourists consume only services. The rate of interest, $r^*$, is fixed in international market. Capital depreciates at a constant exponential rate, $\delta_k$. The households hold wealth and land and receive income from wages, land rent, and interest payments of wealth. Land is only for residential and service use. Technologies of the production sectors are described by the Cobb-Douglas production functions. All markets are perfectly competitive and capital and labor are completely mobile between the three sectors. Capital is perfectly mobile in international market and we neglect possibility of emigration or/and immigration. We assume that labor is homogeneous and is fixed. We assume that the economy is too small to affect the interest rate in the world market. Let $\bar{N}$ stand for the population and $T(t)$ stand for the work time of the representative household. The labor force $N(t)$ is given by

$$N(t) = hT(t)\bar{N}, \tag{1}$$

where $h$ is the fixed level of human capital. We use subscript index, $i$, $s$, and $p$, to denote respectively the industrial, service, and public sectors. Let $K_j(t)$ and $N_j(t)$ stand for the capital stock and labor force employed by sector $j$, $j = i, s, p$, at time $t$. We introduce

$$k_j(t) \equiv \frac{K_j(t)}{N_j(t)}, \quad j = i, s, e, \quad r_\delta \equiv r^* + \delta_k, \quad \bar{\tau}_j \equiv 1 - \tau_j$$

where $\tau_j$ is the fixed tax rate on sector $j$, $0 < \tau_j < 1$, $j = i, s$.

**Industrial sector**. Production function includes, together with private inputs, public goods, externalities, and congestion. Output depends on inputs of private and public capital. Supply of public services introduces a positive externality in production so that the complete production is one of overall increasing returns to scale in the productive factors. Basing on Eicher and Turnovsky (2000), we specify the production function of the industrial sector $F_i(t)$ as follows

$$F_i(t) = \Omega_i(t) K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \quad \alpha_i + \beta_i = 1, \quad \alpha_i, \beta_i > 0, \tag{1}$$

where $\alpha_i$ and $\beta_i$ are parameters and $\Omega_i(t)$ is a function of externalities, public service and congestion. We specify $\Omega_i(t)$ as follows

$$\Omega_i(t) = A_i G^{\theta_p}(t) K_i^{\theta_e}(t) \left( \frac{K_p(t)}{K_i(t) + K_s(t)} \right)^{\theta_c}, \quad \theta_p, \theta_e, \theta_c \geq 0, \tag{2}$$

where $G^{\theta_p}(t)$ measures the effect of public service on productivity, $K_i^{\theta_e}(t)$ the effect of externalities, and $\left(K_p(t)/K_i(t)\right)^{\theta_c}$ the effect of congestion of public goods. Similar to Eicher and Turnovsky (2000), we interpret that when $\theta_e = \theta_c = 0$, there is no congestion and no externality. The nonrival and nonexcludable public service is available equally to each agent, independent of the usage of others. Obviously this is a limited case as most of public services are subject to some degree of congestion. We take account of congestion effects by the term, $\left(K_p(t)/K_i(t)\right)^{\theta_c}$, implies that for a fixed level of public capital, a rise in the private capital tends to reduce the efficiency of public services. There are different ways of describing congestion (see, Gómez, 2008). It should be noted that $G(t)$ is often interpreted to be generated by learning-by-doing or human capital spill-over effects. We now interpret the variables as public goods such as physical and institutional infrastructures. The aggregate public goods $G(t)$ is supplied by the government and is taken as given by the firms. Despite increasing social returns to scale, the function allows to maintain the assumption of perfect competition in the goods market since the technology exhibits constant returns to scale for any given level of public goods, which firms cannot control. It is reasonable to consider that production efficiency will be improved as service level of the public sector is improved, it seems that doubling the service level will not double the output with fixed $K_i(t)$ and $N_i(t)$. That is, the parameter $\theta_p$ should be less than one. Because public service is freely available to firms, it is not a decision variable for the industrial sector.

The government finances the public sector by imposing taxes on, for instance, the outputs of the firms, the capital income, the labor income, and consumptions. We assume that the government imposes taxes on firms' output. Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. The wage rate, $w(t)$, is determined in domestic market. The marginal conditions are

$$r_\delta = \alpha_i \,\bar{\tau}_i \,\Omega_i(t) k_i^{-\beta_i}(t), \quad w(t) = \beta_i \,\bar{\tau}_i \,\Omega_i(t) k_i^{\alpha_i}(t). \tag{3}$$

**Service sector**. The service sector uses three inputs, capital $K_s(t)$, labor force $N_s(t)$, and land $L_s(t)$, to supply services. The production function of the service sector is

$$F_s(t) = A_s K_s^{\alpha_s}(t) N_s^{\beta_s}(t) L_s^{\gamma_s}(t), \quad \alpha_s, \beta_s, \gamma_s > 0, \quad \alpha_s + \beta_s + \gamma_s = 1, \tag{4}$$

where $A_s$, $\alpha_s$, $\beta_s$, and $\gamma_s$ are parameters. Let $p(t)$ and $R(t)$ stand respectively for the price of the service and the land rent. In this study we assume that the prices are determined by market mechanism. The marginal conditions for the service sector are

$$r_\delta = \alpha_s \,\bar{\tau}_s \,A_s \,p(t) k_s^{\alpha_s - 1}(t) l_s^{\gamma_s}(t), \quad w(t) = \beta_s \,\bar{\tau}_s \,A_s \,p(t) k_s^{\alpha_s}(t) l_s^{\gamma_s}(t),$$
$$R(t) = \gamma_s \,\bar{\tau}_s \,A_s \,p(t) k_s^{\alpha_s}(t) l_s^{\gamma_s - 1}(t), \tag{5}$$

where $l_s(t) \equiv L_s(t)/N_s(t)$.

**Full employment of capital and labor**. The total capital stocks employed by the country, $K(t)$, is used by the three sectors. As full employment of capital is assumed, we have

$$K_i(t) + K_s(t) + K_p(t) = K(t). \tag{6}$$

For the labor market we have

$$N_i(t) + N_s(t) + N_p(t) = N(t). \tag{7}$$

**Behavior of domestic households**. We now model behavior of households. We use $L$ to denote the total land available for residential and service use. Each household gets income from land ownership, wealth and wage. Land properties may be distributed in multiple ways under various institutions. This study assumes that the land is equally owned by the population. This implies that the revenue from land is equally shared among the population. Each household gets the land income

$$\bar{r}(t) = \frac{L\,R(t)}{\bar{N}}. \tag{8}$$

Consumers make decisions on lot size, consumption levels of industrial goods and services as well as on how much to save. This study uses the approach to consumers' behavior proposed by Zhang (1993). The current income of the typical household is

$$y(t) = \hat{\tau}_k\, r^*\, \bar{k}(t) + \hat{\tau}_w\, hT(t)w(t) + \hat{\tau}_L\, \bar{r}(t), \tag{9}$$

where $r^*\, k(t)$ is the interest payment, $hT(t)w(t)$ the total wage income, and

$$\hat{\tau}_m \equiv 1 - \tilde{\tau}_m, \quad m = k,\, w,\, L,$$

where $\tilde{\tau}_k$, $\tilde{\tau}_w$ and $\tilde{\tau}_L$ are respectively the fixed tax rates on the wealth (excluding land) income, wage, and land income. We call $y(t)$ the current (disposable) income in the sense that it comes from consumers' wages and current earnings from ownership of wealth. The disposable income at any point in time is

$$\hat{y}(t) = y(t) + \bar{k}(t). \tag{10}$$

The disposable income is used for saving and consumption. At time $t$ the consumer has the total amount of income equalling $\hat{y}(t)$ to distribute between consuming and saving. In the growth literature, for instance, in the Solow model, the saving is out of the current income, $y(t)$, while in this study the saving is out of the disposable income which is dependent both on the current income and wealth. The representative household's budget constraint is

$$(1 + \tilde{\tau}_R)R(t)l(t) + (1 + \tilde{\tau}_s)\,p(t)c_s(t) + (1 + \tilde{\tau}_i)c_i(t) + s(t) = \hat{y}(t), \tag{11}$$

where $\tilde{\tau}_R$, $\tilde{\tau}_R$, and $\tilde{\tau}_R$ are respectively the consumer tax rates on housing, consumption of services, and consumption of goods. Equation (12) means that the consumption and saving exhaust the consumers' disposable income. Let $\bar{T}(t)$ stand for the leisure time at time $t$. The time constraint is

$$T(t) + \bar{T}(t) = T_0, \tag{12}$$

where $T_0$ is the total time available for work and leisure. Substituting (13) into (12) yields

$$\hat{\tau}_w\, h\bar{T}(t)w(t) + (1 + \tilde{\tau}_R)R(t)l(t) + (1 + \tilde{\tau}_s)\,p(t)c_s(t) + (1 + \tilde{\tau}_i)c_i(t) + s(t) = \bar{y}(t), \tag{13}$$

where

$$\bar{y}(t) = (1 + \hat{\tau}_k\, r^*)\bar{k}(t) + \hat{\tau}_w\, hT_0\, w(t) + \hat{\tau}_L\, \bar{r}(t). \tag{14}$$

The utility function $U(t)$ is dependent on $\bar{T}(t)$, $l(t)$, $c_s(t)$, $c_i(t)$ and $s(t)$ as follows

$$U(t) = \theta(G(t))\bar{T}^{\sigma_0}(t) l^{\eta_0}(t) c_s^{\gamma_0}(t) c_i^{\xi_0}(t) s^{\lambda_0}(t), \ \ \sigma_0, \eta_0, \gamma_0, \xi_0, \lambda_0 > 0,$$

in which $\sigma_0$, $\eta_0$, $\gamma_0$ $\xi_0$ and $\lambda_0$ are the representative household's elasticity of utility with regard to leisure time, lot size, services, industrial goods, and saving, and $\theta(G(t))$ is the amenity which is related to the level of public good. We call $\sigma_0$, $\eta_0$, $\gamma_0$ $\xi_0$ and $\lambda_0$ propensities to consume the leisure time, to use the lot size, to consume services, to consume industrial goods, and to hold wealth, respectively. Maximizing $U(t)$ subject to the budget constraint yields

$$\bar{T}(t) = \frac{\sigma \bar{y}(t)}{w(t)}, \ \ l(t) = \frac{\eta \bar{y}(t)}{R(t)}, \ \ c_s(t) = \frac{\gamma \bar{y}(t)}{p(t)}, \ \ c_i(t) = \xi \bar{y}(t), \ \ s(t) = \lambda \bar{y}(t), \tag{15}$$

where

$$\sigma \equiv \frac{\rho \sigma_0}{\hat{\tau}_w h}, \ \ \eta \equiv \frac{\rho \eta_0}{1 + \tilde{\tau}_R}, \ \ \gamma \equiv \frac{\rho \gamma_0}{1 + \tilde{\tau}_s}, \ \ \xi \equiv \frac{\rho \xi_0}{1 + \tilde{\tau}_i}, \ \ \lambda \equiv \rho \lambda_0,$$
$$\rho \equiv \frac{1}{\sigma_0 + \eta_0 + \gamma_0 + \xi_0 + \lambda_0}.$$

According to the definition of $s(t)$ the wealth accumulation for the household is

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t). \tag{16}$$

**Demand function of foreign tourists**. Let $y_f(t)$ stand for the disposable income of foreign countries. According to Schubert and Brida (2009), we assume the following iso-elastic tourism demand function

$$D_T(t) = a(t) y_f^{\phi}(t) [(1 + \tilde{\tau}_s) p(t)]^{-\varepsilon}, \tag{17}$$

where $\phi$ and $\varepsilon$ are respectively the income and price elasticities of tourism demand. The variable, $a(t)$, is dependent on many conditions, such as infrastructures such as airports, transportation systems and travel costs (e.g., Ivanovic, Baresa, and Bogdan, 2014) and social environment (like criminal rates and traffic congestions), and cultural capital (e.g., Throsby, 1999; Beerli and Martin, 2004). We assume that tourists pay the same price in consumption as domestic people. The amenity is a factor of attractiveness of the small country. It is obviously that people would love to travel a place with good environment. In reality, tourism industry has many special features which have important effects on pricing (e.g., Marin-Pantelescu and Tigu, 2010; Stabler, Papatheodorou and Sinclair, 2010). This study tries to make tourism part as simple as possible as the system is already very complicated.

**Full use of land**. The available land is fully used for housing and service production. This implies

$$l(t)\bar{N} + L_s(t) = L. \tag{18}$$

**Demand and supply for services**. The equilibrium condition for services is

$$c_s(t)\bar{N} + D_T(t) = F_s(t). \tag{19}$$

**Behaviour of the government**. Following Zhang (2014), we now describe the public sector. The government financially supports the public sector. The capital stocks and workers employed by the public sector are paid at the same rates that the private sectors pay the services of these factors. We assume that all the tax incomes are spent by the public sector. Our study takes account of tax not only on producers, but also on consumers. The government's tax incomes consist of the incomes on the production sectors, consumption, wage income and wealth income. The government's income is given by

$$Y_p(t) = \tau_i F_i(t) + \tau_s \, p(t) F_s(t) + I_h(t) \bar{N} + \tilde{\tau}_T \, p(t) D_T(t), \tag{20}$$

where the tax income from the representative household is

$$I_h(t) = \tilde{\tau}_R R(t) l(t) + \tilde{\tau}_i c_i(t) + \tilde{\tau}_s \, p(t) c_s(t) + \tilde{\tau}_k r^* \bar{k}(t) + \tilde{\tau}_L \bar{r}(t) + \tilde{\tau}_w h w(t) T(t).$$

To determine how the public sector decides the number of labor force and the level of capital, we assume that the public sector behaves effectively in the sense that it uses the available resource to maximize public services. We assume that the public sector supplies public goods by utilizing capital, $K_p(t)$, and labor force, $N_p(t)$, as follows

$$G(t) = A_p K_p^{\alpha_p}(t) N_p^{\beta_p}(t), \quad A_p, \alpha_p, \beta_p > 0.$$

For the given tax rates, the public sector is faced with the budget constraint

$$w(t) N_p(t) + r_\delta K_p(t) = Y_p(t). \tag{21}$$

Maximizing public services under the budget constraint yields

$$w(t) N_p(t) = \alpha Y_p(t), \quad r_\delta K_p(t) = \beta Y_p, \tag{22}$$

where

$$\alpha \equiv \frac{\alpha_p}{\alpha_p + \beta_p}, \quad \beta \equiv \frac{\beta_p}{\alpha_p + \beta_p}.$$

We have thus built the dynamic growth model with endogenous wealth, public goods, and tourism. The model is a synthesis of the Solow-Uzawa growth models and neoclassical growth models with public goods for a small open economy with tourism. We now examine the behavior of the model.

## 2. SIMULATING MOTION OF THE NATIONAL ECONOMY

We now show that the motion of the economic system is described by two nonlinear differential equations. The following lemma shows how we can determine the motion of all the variables in the dynamic system.

**Lemma**. The motion of the land rent and environment is determine by the following two differential equations

$$\dot{R}(t) = \Omega_1(R(t), w(t)), \quad \dot{w}(t) = \Omega_2(R(t), w(t)), \tag{23}$$

where $\Omega_1$ and $\Omega_2$ are functions of $R(t)$ and $w(t)$ determined in the appendix. By the following procedure we can determine all the variables as functions of $R(t)$ and $w(t)$: $k_i(t)$ and $\Omega_i(t)$ by (A1) $\rightarrow$ $k_s(t)$ by (A2) $\rightarrow$ $k_p(t)$ by (A3) $\rightarrow$ $l_s(t)$ by (A5) $\rightarrow$ $p(t)$ by (A10) $\rightarrow$ $D_T(t)$ by (A11) $\rightarrow$ $N_s(t)$ by (A15) $\rightarrow$ $K_s(t) = k_s(t) N_s(t)$ $\rightarrow$ $\bar{k}(t)$ by (A16) $\rightarrow$ $\bar{y}(t)$ by (A7) $\rightarrow$ $\bar{T}(t)$, $l(t)$, $c_s(t)$, $c_i(t)$, $s(t)$ by (15) $\rightarrow$ $T(t) = T_0 - \bar{T}(t)$ $\rightarrow$

$N(t) = hT(t)\bar{N} \rightarrow \bar{r}(t)$ by (8) $\rightarrow K(t)$ by (A22) $\rightarrow N_i(t)$ and $N_e(t)$ by (A19) $\rightarrow$ $K_m(t) = k_m(t)N_m(t),\ m = i,\ p \rightarrow L_s(t) = l_s(t)N_s(t) \rightarrow F_i(t)$ by (1) $\rightarrow F_s(t)$ by (4) $\rightarrow$ $G(t) = A_p K_p^{\alpha_p}(t) N_p^{\beta_p}(t) \rightarrow Y_p(t)$ by (A20).

The lemma implies that for a given rate of interest in the global market, the economic system at any point in time can be uniquely described as functions of the land rent and wage rate. Hence, if we know the motion of the land rent and wage rate, we can determine the motion of the whole system. It should be remarked that in the small open growth model Turnovsky (1996) shows that the equilibrium growth rates of domestic capital and consumption are determined largely independent. According to Turnovsky domestic capital is determined by production conditions, and consumption is determined primarily by tastes (see also Zeira 1987). In our model, the total output levels, the capital stocks employed by the economy, and economic production structure are not only determined by the production conditions and the internationally fixed rate of interest, but also by tastes. Moreover, consumption is not only determined by preferences but also related to the rate of interest and the production conditions.

As the expressions of our result are tedious, it is difficult to get explicit conclusions. For interpretation, we simulate the model. We specify parameter values as follows

$$r^* = 0.04,\ \delta_k = 0.05,\ \bar{N} = 20,\ h = 2,\ T_0 = 24,\ L = 8,\ A_i = 1.1,\ A_s = 1.4,\ A_p = 0.5,$$
$$\alpha_i = 0.33,\ \alpha_s = 0.25,\ \beta_s = 0.65,\ \alpha_p = 0.3,\ \beta_p = 0.7,\ \lambda_0 = 0.7,\ \xi_0 = 0.15,\ \gamma_0 = 0.06,$$
$$\eta_0 = 0.06,\ \sigma_0 = 0.2,\ a = 1,\ y_f = 4,\ \phi = 1.5,\ \varepsilon = 1.6,\ \theta_p = 0.1,\ \theta_e = \theta_c = 0.05,$$
$$\tau_i = \tau_s = \tau_c = \tau_k = \tau_w = 0.01,\ \bar{\tau}_k = \bar{\tau}_w = \bar{\tau}_L = \bar{\tau}_R = \bar{\tau}_s = \bar{\tau}_i = 0.01,\ \theta_i = 0.1,\ \theta_s = 0.05,$$
$$\theta_s = 0.05,\ \tau_i = \tau_s = \tau_c = \tau_k = \tau_w = 0.01,\ \bar{\tau}_k = \bar{\tau}_w = \bar{\tau}_L = \bar{\tau}_R = \bar{\tau}_s = \bar{\tau}_i = 0.01,$$
$$\tilde{\theta}_i = 0.05,\ \tilde{\theta}_s = 0.01,\ \tilde{\theta}_T = 0.05,\ \theta_0 = 0.05,\ b_i = b_s = 0.15,\ b_e = 0.1,\ b_0 = 0.05. \tag{21}$$

The rate of interest is fixed at 3 per cent and the population is 20. Many empirical studies use the value of the parameter, $\alpha$, in the Cobb-Douglas production functions approximately 0.3. Some empirical studies show that income elasticity of tourism demand is well above unity (Syriopoulos 1995; Lanza, Temple, and Urga 2003). According to Lanza *et al.* (2003), the price elasticity is in the range between 1.03 and 1.82 and income elasticities are in the range between 1.75 and 7.36. Refer to, for instance, Gaŕin-Mũnos (2007) for other studies on elasticities of tourism. Tax rates are fixed at 1 or 0.5 percent. We assume relatively weak effects of public goods, externalities and congestions. We specify the initial conditions as follows

$$R(0) = 30,\ w(0) = 1.9.$$

We plot the motion of the dynamic system in Figure 1. As their initial values are fixed lower than their long-term equilibrium values, the land rent and wage rise over time. In tandem with rising land rent, the price of services is enhanced. Rising price reduces tourist demand. In association with rising wage rate and wealth the leisure time is increased. The total labor supply falls. The GDP falls slightly. The government gets more money and spends more on supplying public goods. The public produces more and employs more capital and labor inputs. The output level of the industrial sector is reduced and that of the service sector is increased. The labor and capital inputs of the service sectors are increased and the labor and capital inputs are reduced. The national wealth rises over time and the capital stocks employed by the country falls. The household consumes more industrial goods and services, owns more wealth, and has larger lot size.

Figure 1 shows the motion of the variables over time. From the figure we observe that all the variables of the economic system tend to become stationary in the long term. This implies that the system approaches an equilibrium point. We identify the equilibrium values of the variables as follows

$$w = 1.744,\ p = 1.911,\ R = 35.9,\ Y = 1426.8,\ Y_p = 27.65,\ N = 451.4,\ K = 4090.1,$$
$$\bar{K} = 3073.8,\ DT = 2.79,\ N_i = 342.1,\ N_s = 98.2,\ N_p = 11.1,\ K_i = 3265.9,\ K_s = 732,$$
$$K_p = 92.2,\ L_s = 0.73,\ F_i = 899.7,\ F_s = 139.2,\ G = 10.47,\ \bar{T} = 12.7,\ c_i = 32.6,$$
$$c_s = 6.82,\ l = 0.36,\ \bar{k} = 153.7.$$

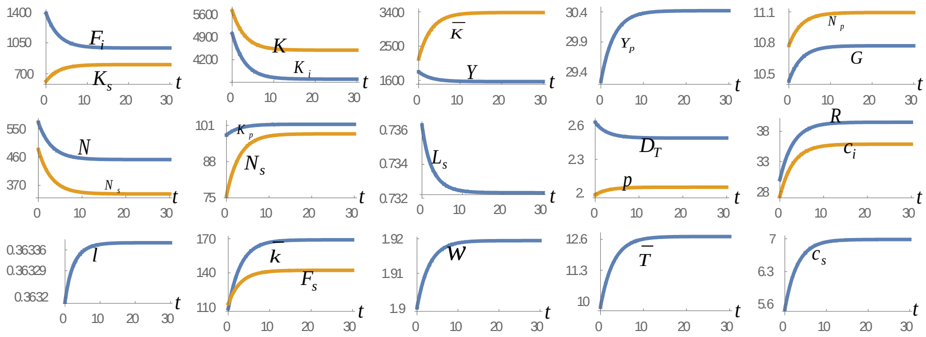

**Figure 1.** The motion of the national economy

The eigenvalues are

$$\{-0.332,\ 0\}.$$

As the dynamical system is genuinely one-dimensional as demonstrated in the appendix, this confirms that the unique equilibrium point is stable. This result is important as it also guarantees the validity of comparative dynamic analysis in the next section.

## 3. COMPARATIVE DYNAMIC ANALYSIS

The previous section plots the motion of the variables. This section examines how changes in some parameters affect the national economy over time. As we have shown how to simulate the motion of the system, it is straightforward to make comparative dynamic analysis. We introduce a variable, $\bar{\Delta} x(t)$, to stand for the change rate of the variable, $x(t)$, in percentage due to changes in the parameter value.

**A rise in the rate of interest in the global market.** First, we study the effects of changes in the rate of interest $r^*$ on the national economy. The rate of interest is changed as follows: $r^* = 0.04 \Rightarrow 0.05$. It should be remarked that as we have explicitly given the procedure to follow the motion of the economy system, we can also carry out comparative dynamic analysis by assuming that the rate of interest varies in time, $r^*(t)$.

This is true also for other parameters. The effects are plotted in Figure 2. In the rest of the paper, a solid (dashed) line in a plot demonstrates the value before (after) a parameter is changed. As the cost of capital is increased, the wage rate is increased. The economy employs less capital and the GDP is slightly reduced. The household's and national wealth are increased. Although the public sector gets more money, the public sector's output falls due to the rising costs of the two input factors. The two inputs of the public sector are decreased. The rise in the cost of capital causes the two sectors to use less capital. The output level and labor input of the industrial sector are reduced. The service sector's output is reduced and its labor input is increased. Less foreign tourists visit the country in tandem with rising price of services. The land rent is increased. The service sector uses less land and the lot size is expanded. The leisure time rises in association with rising wage rate (i.e., opportunity cost of leisure). The household consumes less services and more industrial goods.

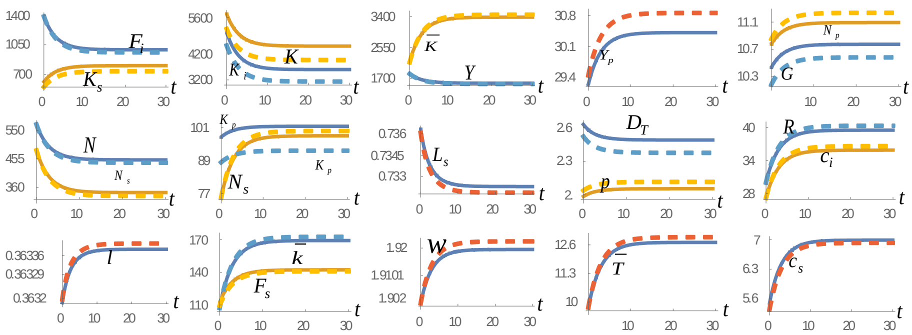

**Figure 2.** A rise in the rate of interest

**A rise in the total productivity of the service sector**. We now examine the impact of the following change in the total productivity of the service sector: $A_s = 1.4 \Rightarrow 1.5.$ We plot the effects on the variables in Figure 3. The increased productivity of the service sector raises the output of services and lowers the price of services. More foreign tourists are attracted to the country. The leisure time, total labor supply, the total capital, the national wealth, the GDP, the public sector, and the output of the industrial sector are slightly affected. The land use is redistributed, the lot size being reduced. The household's consumption of services is increased in association with falling price of services. We see that changes in the service sector's productivity mainly affect services-related activities and have weak effects on the nationally aggregated real variables.

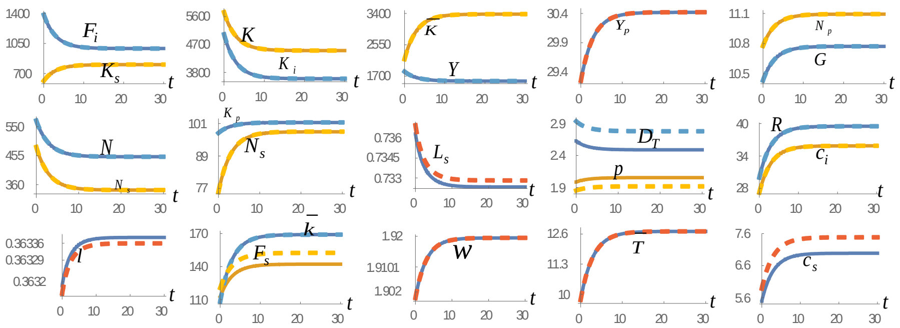

**Figure 3.** A rise in the total productivity of the service sector

**Raising tax rate on the service sector**. We now study the effects of raising the tax rate on the service sector in the following way: $\tau_s = 0.01 \Rightarrow 0.02$. The effects are plotted in Figure 4. As the tax rate is increased, the government gets more income. The public sector employs more capital and labor inputs and supplies more public goods. The labor input of the service sector is reduced and the labor input of the industrial sector is slightly affected. The household's wealth and national wealth are slightly augmented. The national economy employs more capital. Less foreign tourists visit the country in tandem with rising price of services. The wage rate, leisure time, and total labor supply are slightly affected. The industrial sector produces more and employs more capital. The land rent is increased. The service sector uses less land and the lot size is expanded. The household consumes less services and more industrial goods. The GDP is slightly reduced.

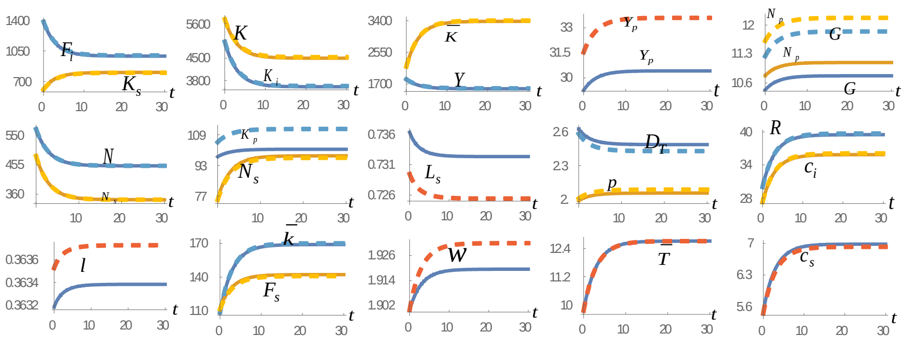

**Figure 4.** Raising tax rate on the service sector

**Raising tax rate on consumption of services**. We now study the effects of raising the tax rate on consumption of services in the following way: $\tilde{\tau}_s = 0.01 \Rightarrow 0.02$. The effects are plotted in Figure 5. Rather than raising taxes on producers, the government gets more income from consumers. Nevertheless, by comparing Figure 4 and Figure 5, we see that the effects on the economy are almost the same by two different tax policies.

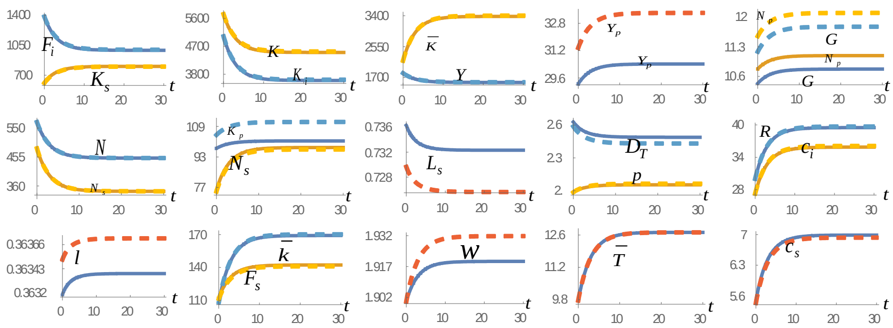

**Figure 5.** Raising tax rate on consumption of services

**An improvement in human capital**. We now examine what will happen to the national economy when human capital is improved as follows: $h = 2 \Rightarrow 2.2$. The changes in the variables are plotted in Figure 6. The leisure time is slightly affected and the total labor supply is increased in tandem with rising wage rate. The GDP and the total capital employed by the economy are increased. The household's wealth and national wealth are augmented. The price of services and the land rent are increased. The increased price reduces foreign tourists. The lot size is increased. The land use of the service sector is reduced. The household consumes more goods and services and owns more wealth. Each sector increases their inputs and output level. The government gets more money for supplying public goods. The government sector's output and its two inputs are augmented.

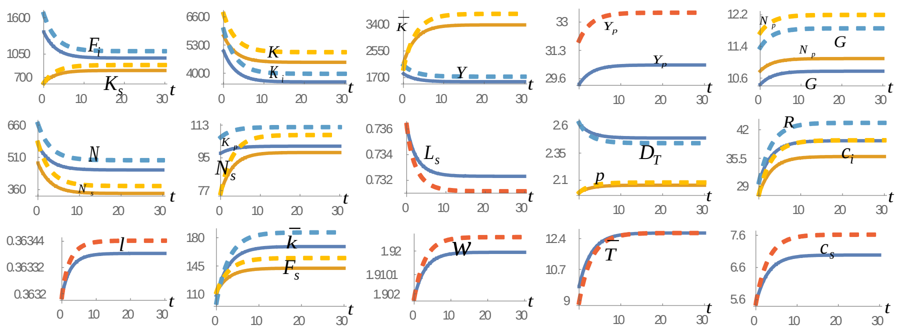

**Figure 6.** An improvement in human capital

**A rise in the household's propensity to consume services**. We now examine what will happen to the national economy when the household's propensity to consume services is augmented as follow: $\gamma_0 = 0.06 \Rightarrow 0.08$. The changes in the variables are plotted in Figure 7. The consumption level of services by the domestic households is increased. The price and tourism are slight affected. The consumption level of industrial goods falls in association with rising consumption level of the household. The leisure time is increased and the total labor supply is increased in tandem with rising wage rate. The GDP and the total capital employed by the economy are increased. The household's wealth and national wealth are lowered. The land rent is reduced. The lot size is decreased. The land use of the service sector is augmented. The service sector increases the two inputs and output level. The industrial sector decreases the two inputs and output level. The government gets more money for supplying public goods. The government sector's output and its two inputs are augmented.

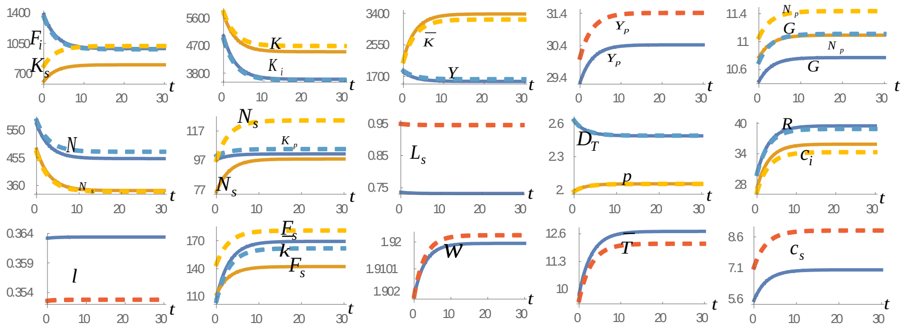

**Figure 7.** A rise in the household's propensity to consume services

**Public services more strongly affecting the productivity of the industrial sector**. We now allow public services to more strongly affect the productivity of the industrial sector in the following way: $\theta_p = 0.1 \Rightarrow 0.2$. The changes in the variables are plotted in Figure 8. The wage rate and output level of the industrial sector are increased. The industrial sector also employs more capital input. The leisure time, total labor supply, labor inputs of the industrial and service sectors are slightly affected. The GDP and the capital employed by the economy are augmented. The price is increased and tourism is reduced. The land rent is enhanced and the lot size is expanded. The government gets more money for supplying public goods. The government sector's output and its two inputs are augmented.

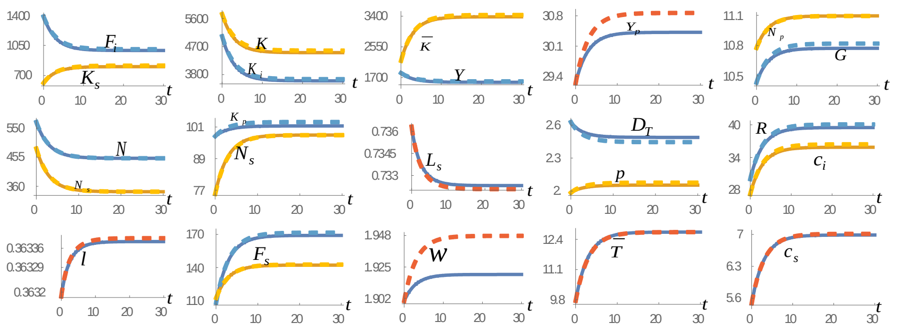

**Figure 8.** Public services more strongly affecting the productivity

## CONCLUSION

This paper built a growth model of a small open economy with tourism and endogenous wealth and public goods in a perfectly competitive economy. The national economy consists of three — service, industrial and public — sectors. The small-open economy implies that the rate of interest is fixed in international market. The production side is the same as in the neoclassical growth theory. We used a utility function proposed by Zhang (1993) to determine saving, consumption and time distribution. We simulated the motion of the model and examined effects of changes in the rate of interest, the price elasticity of tourism, the government's tax rates on the service sector and consumption of services, the total productivity of the service sector, the propensity to consume services, and the impact of public services on the productivity of the industrial sector. The comparative dynamic analysis provides some important insights. It should be remarked that the model can be extended and generalized in different directions. We may study the economic dynamics when utility and production functions are taken on other functional forms. It is also important to generalize model to include the case that domestic households travel to other countries. It is necessary deal with economies as an integrated whole (Morley, Rosselló, and Santana-Gallego 2014). Monetary issues such as exchange rates and inflation policies are important for understanding trade issues.

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## APPENDIX: PROVING THE LEMMA

From (3) we have

$$k_i(w) = \frac{\alpha_i w}{\beta_i r_\delta}, \quad \Omega_i(w) = \frac{w}{\beta_i \bar{\tau}_i k_i^{\alpha_i}}. \tag{A1}$$

where we omit time variable in expressions. Hence, we can treat $k_i$ and $\Omega_i$ as functions of $w$. From (5) we solve

$$k_s(w) = \frac{\alpha_s w}{\beta_s r_\delta}. \tag{A2}$$

We consider $k_s$ as a function of $w$. From (22) we have

$$k_p(w) = \frac{\alpha w}{\beta r_\delta}. \tag{A3}$$

From (6) and (7) and the definitions of $k_j$

$$k_i N_i + k_s N_s + k_e N_e = K, \quad N_i + N_s + N_e = N. \tag{A4}$$

From (5), we solve

$$l_s(R, w) = \frac{w \gamma_s}{\beta_s R}. \tag{A5}$$

Insert (A5) in (18)

$$l \bar{N} + \frac{w \gamma_s N_s}{\beta_s R} = L. \tag{A6}$$

From the definition of $\bar{y}$, we have

$$\bar{y} = \tau^* \bar{k} + \hat{\tau}_w h T_0 w + \frac{\hat{\tau}_L L R}{\bar{N}}, \tag{A7}$$

where $\tau^* \equiv 1 + \hat{\tau}_k r^*$. From (A7) and $l = \eta \bar{y} / R$ in (15)

$$\frac{l}{\eta} = \frac{\tau^* \bar{k} + \hat{\tau}_w h T_0 w}{R} + \frac{\hat{\tau}_L L}{\bar{N}}. \tag{A8}$$

Insert (A8) in (A6)

$$\bar{k} + \hat{\tau}_0 N_s = \hat{\tau}_1, \tag{A9}$$

where

$$\hat{\tau}_1(R, w) \equiv \left(\frac{1}{\eta} - \hat{\tau}_L\right) \frac{L R}{\tau^* \bar{N}} - \frac{\hat{\tau}_w h T_0 w}{\tau^*}, \quad \hat{\tau}_0(w) \equiv \frac{w \gamma_s}{\tau^* \beta_s \eta \bar{N}}.$$

From (5)

$$p(R, w) = \frac{R}{\gamma_s \bar{\tau}_s A_s k_s^{\alpha_s} l_s^{\gamma_s - 1}}. \tag{A10}$$

By (16) and (A11)

$$D_T(R, w) = a\, y_f^{\phi} \left[\left(1 + \tilde{\tau}_s\right)p\right]^{-\varepsilon}. \tag{A11}$$

From $r_\delta = \alpha_s \bar{\tau}_s\, p F_s / K_s$ and (18) we have

$$c_s \bar{N} + D_T = \frac{r_\delta K_s}{\bar{\tau}_s \alpha_s\, p}. \tag{A12}$$

Insert $c_s = \gamma \bar{y} / p$ in (A12)

$$\gamma \bar{y} \bar{N} + p D_T = \frac{r_\delta K_s}{\bar{\tau}_s \alpha_s}. \tag{A13}$$

Insert (A7) in (A13)

$$\tau^* \bar{N} \bar{k} + \hat{\tau}_w h T_0 \bar{N} w + \hat{\tau}_L L R + \frac{p D_T}{\gamma} = \frac{r_\delta K_s}{\bar{\tau}_s \gamma \alpha_s}. \tag{A14}$$

Insert (A9) in (A14)

$$N_s(R, w) = \left(\tau^* \bar{N} \hat{\tau}_1 + \hat{\tau}_w h T_0 \bar{N} w + \hat{\tau}_L L R + \frac{p D_T}{\gamma}\right)\left(\frac{1}{\bar{\tau}_s \gamma} + \frac{\gamma_s}{\eta}\right)^{-1} \frac{\beta_s}{w}, \tag{A15}$$

where we also use $K_s = k_s N_s$. Hence we have $K_s(R, w) = k_s N_s$.
By (A9) and (A15)

$$\bar{k} = \Lambda(R, w) \equiv \hat{\tau}_1 - \hat{\tau}_0 N_s. \tag{A16}$$

By (A7) we have $\bar{y}(R, w)$. From the results so far, (15), (8), the time constraint and (1), we solve

$$\bar{T},\ l,\ c_s,\ c_i,\ s,\ \bar{r},\ T,\ N$$

as functions of $R$ and $w$. From its definition and the results so far, we have

$$I_h(R, w) = \tilde{\tau}_R R l + \tilde{\tau}_i c_i + \tilde{\tau}_s p c_s + \tilde{\tau}_k r^* \bar{k} + \tilde{\tau}_L \bar{r} + \tilde{\tau}_w h w T. \tag{A17}$$

By (A4)

$$k_i N_i + k_p N_p = K - k_s N_s, \quad N_i + N_p = N - N_s. \tag{A18}$$

Solve (A18)

$$N_i = f_i - k_0 K, \quad N_p = k_0 K + f_p, \tag{A19}$$

where

$$k_0(R, w) \equiv \frac{1}{k_p - k_i} = \frac{\alpha_0}{w}, \quad \alpha_0 \equiv \frac{r_\delta}{\alpha / \beta - \alpha_i / \beta_i},$$

$$f_i(R, w) \equiv \frac{\alpha_0 \alpha N}{\beta r_\delta} - \left(\frac{\alpha}{\beta} - \frac{\alpha_s}{\beta_s}\right)\frac{\alpha_0 N_s}{r_\delta}, \quad f_p(R, w) \equiv -\frac{\alpha_0 \alpha_i N}{\beta_i r_\delta} + \left(\frac{\alpha_i}{\beta_i} - \frac{\alpha_s}{\beta_s}\right)\frac{\alpha_0 N_s}{r_\delta}.$$

From (20), (3) and (5), we have

$$Y_p = \tau_i A_i \Omega_i k_i^{\alpha_i} N_i + \tilde{I}_h, \tag{A20}$$

where we also use

$$w = \frac{\bar{\tau}_i \beta_i F_i}{N_i} = \frac{\bar{\tau}_s \beta_s p F_s}{N_s}, \quad \tilde{I}_h(R, w) \equiv \tau_s \frac{w N_s}{\bar{\tau}_s \beta_s} + I_h \bar{N} + \tilde{\tau}_T p D_T.$$

Insert (22) in (A20)

$$\frac{w N_p}{\beta} = \frac{\tau_i N_i w}{\beta_i \bar{\tau}_i} + \tilde{I}_h. \tag{A21}$$

Insert (A19) in (A21)

$$K(R, w) = \left( \frac{\tau_i f_i w}{\beta_i \bar{\tau}_i} + \tilde{I}_h - \frac{f_p w}{\beta} \right) \left( \frac{1}{\beta} + \frac{\tau_i}{\beta_i \bar{\tau}_i} \right)^{-1} \frac{1}{w k_0}. \tag{A22}$$

We see that by the procedure in the lemma we can determine all the variables as functions of $R$ and $w$. From the procedure and (17), we have

$$\dot{\bar{k}} = \Omega_0(R, w) \equiv s - \bar{k}, \tag{A23}$$

Taking derivatives of (A16) with respect to time yield

$$\dot{\bar{k}} = \frac{\partial \Lambda}{\partial R} \dot{R} + \frac{\partial \Lambda}{\partial w} \dot{w}. \tag{A24}$$

From (A1) and (2) we have

$$\Omega_i = \bar{\delta} w^{\beta_i} = \bar{\Lambda}(R, w) \equiv A_i G^{\theta_p} K_i^{\theta_e} \left( \frac{K_p}{K_i} \right)^{\theta_c}. \tag{A25}$$

where

$$\bar{\delta} \equiv \left( \frac{\alpha_i}{\beta_i r_\delta} \right)^{-\alpha_i} \frac{1}{\beta_i \bar{\tau}_i}.$$

Taking derivatives of (A25) with respect to time yield

$$\dot{w} = \bar{\Omega} \dot{R}, \tag{A26}$$

where

$$\bar{\Omega}(R, w) \equiv \left( \bar{\delta} \beta_i w^{-\alpha_i} - \frac{\partial \bar{\Lambda}}{\partial w} \right)^{-1} \frac{\partial \bar{\Lambda}}{\partial R}.$$

Insert (A23) in (A24)

$$\dot{R} = \Omega_1(R, w) \equiv \left( \frac{\partial \Lambda}{\partial R} + \bar{\Omega} \frac{\partial \Lambda}{\partial w} \right)^{-1} \Omega_0. \tag{A27}$$

where we also use (A26). From (A27) and (A26) we have

$$\dot{w} = \Omega_2(R, w) \equiv \bar{\Omega} \Omega_1. \tag{A28}$$

We do not provide the expressions in the above equations because they are too tedious. We thus proved the lemma.